YIELD LINE ANALYSIS OF MN DOT
BRIDGE RAIL - FIG. 5-397.117.

1. POSSIBLE FRACTURE PATTERNS DUE TO
   GEOMETRY OF BARRIER (SAFETY SHAPE).

   1. FULL SECTION BENDING/YIELDING/Failure.

   2. TOP SECTION YIELDING/Failure.

- SOLVE YIELD LINE FOR BOTH Failures.
1. Full Section

Base width = 18"  
Clear cover (back) = 2 1/4"
\[ d = 18'' - 2\frac{1}{4}'' - 6\frac{1}{2}'' = 15{\frac{3}{8}}'' \]
Stirrup = #5 @ 12"
\[ A_s = A'_s = 0.31 \text{ in}^2 \]

Width of stirrup at deck/barrier interface:
\[ d' = d - 12\frac{3}{8}'' = 3'' \]
\[ f_y = 60 \text{ ksf} \]
\[ f_c = 4 \text{ ksf} \]

Idealized shape:

\[ M_c = 325 \text{ k-in} \]
\[ M_c = 27.1 \text{ k-ft/ft} \]

2. Top Section

Base width = 10\frac{1}{2} + 2\frac{1}{2} = 13"
Clear cover = 2 1/4"
\[ d = 13'' - 2\frac{1}{4}'' - 6\frac{1}{2}'' = 10\frac{3}{8}'' \]

Front cover = 2"  
\[ d' = 2\frac{3}{8}'' \]

Simplified x-section:

\[ A_s = A'_s = 0.31 \text{ in}^2 \]
\[ M_c = 220 \text{ k-in} \]
\[ M_c = 18.3 \text{ k-ft/ft} \]
**WALL MOMENT RESISTANCE**

### Full Section

1. **Ignore toe of barrier (no steel anyway).**

2. **Average width (top):** \(\frac{10.5 + 3}{2} = 11\frac{3}{4}''\)

3. **Average width:** \(\frac{(1\frac{1}{4} \times 22) + (3 \times 12)}{34} = 12\frac{7}{8}''\)

4. **Long steel = #4 bars**

   - **Back row of steel:** \(d = 2\frac{7}{8} + 5\frac{1}{8} + \frac{1}{2}\)

   - **\(d_A = 3\frac{1}{8} + (6 - \frac{5}{8} - \frac{1}{2}) = 8''\)**

   - **Assume even vertical distribution:**
     - \(d_B = 9\frac{7}{8}''\)
     - \(d_c = 10\frac{3}{8}''\)
     - \(d_d = 3\frac{1}{8} + (9\frac{1}{2} - 5\frac{1}{8} - \frac{1}{2}) = 11\frac{1}{2}''\)

### Resistance

<table>
<thead>
<tr>
<th>Section</th>
<th>(A_s = 0.20 \text{ in}^2/\text{bar} )</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(d' = 3\frac{7}{8}'' ) (4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d_A = 8'' ) (1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d_B = 9\frac{7}{8}'' ) (1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d_c = 10\frac{3}{8}'' ) (1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d_d = 11\frac{1}{2}'' ) (1)</td>
<td></td>
</tr>
</tbody>
</table>

**Average:**

- \(M_w = 578 \text{ k-in} \)
- \(M_w = 17 \text{ k-ft/ft} \)

### Resistance

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<thead>
<tr>
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<th>(A_s = 0.20 \text{ in}^2/\text{bar} )</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>(d = 9'' ) (4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d_A = 4\frac{7}{8}'' ) (1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d_B = 3'' ) (1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d_c = 1\frac{3}{4}'' ) (1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d_d = 5\frac{1}{8}'' ) (1)</td>
<td></td>
</tr>
</tbody>
</table>

**Average:**

- \(M_w = 544 \text{ k-in} \)
- \(M_w = 16 \text{ k-ft/ft} \)

**Average:**

- \(M_w = 510 \text{ k-in} \)
- \(M_w = 15 \text{ k-ft/ft} \)
**Mw - CONTINUED.**

**Top Section.**
8-10.5" x 4

\[ \text{Avg Width} = \left(10.5'' + 13\right)/2 = 11\frac{3}{4}'' \]

- Only top 2 rows of steel in top portion. - Same (d)s as previously calculated.

**Resistance**

\[ A_s = 0.20 \text{ in}^2/\text{lin ft} \]
\[ d' = 3\frac{3}{8}'' (2) \]
\[ d_A = 8'' (1) \]
\[ d_B = 9\frac{1}{8}'' (1) \]

**Resist II**

\[ A_k = 0.20 \text{ in}^2/\text{lin ft} \]
\[ d = 8.625'' (2) \]
\[ d_A = 3\frac{3}{4}'' (1) \]
\[ d_B = 2\frac{5}{8}'' (1) \]

\[ M_w = 265 \text{ k-in} \]
\[ M_w = 12.05 \text{ k-ft/ft} \]

\[ M_w = 266.5 \text{ k-in} \]
\[ M_w = 12.1 \text{ k-ft/ft} \]

\[ M_w = 268 \text{ k-in} \]
\[ M_w = 12.18 \text{ k-ft/ft} \]
INTERIOR SECTION

YIELD LINE CALCULATIONS.

\[ L_{cr} = \frac{\ell}{2} + \sqrt{\left(\frac{\ell}{2}\right)^2 + 8H \left(\frac{M_b + M_{wH}}{M_c}\right)} \]

\[ R_w = \frac{8M_b}{L_{cr} - \ell/2} + \frac{8M_{wH}}{L_{cr} - \ell/2} + \frac{M_c L_{cr}^2}{H (L_{cr} - \ell/2)} \]

<table>
<thead>
<tr>
<th>LOAD LENGTH</th>
<th>TEST LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 ft</td>
<td>TL-3</td>
</tr>
<tr>
<td>3.5 ft</td>
<td>TL-4</td>
</tr>
<tr>
<td>8 ft</td>
<td>TL-5</td>
</tr>
</tbody>
</table>

1. FULL SECTION
   \[ M_b = \ell \]
   \[ M_c = 27.1 \text{ k-ft/ft} \]
   \[ M_w = 16.0 \text{ k-ft/ft} \]
   \[ H = 34'' = 2.83 \text{ ft} \]
   \[ \ell = 3.5 \text{ ft} \]
   \[ L_{cr} = 8.15 \text{ ft} \]
   \[ R_w = 155.9 \text{ kips} \]
   \[ & R_w = 140 \text{ kips} \]

2. TOP SECTION
   \[ M_b = 0 \]
   \[ M_c = 18.3 \text{ k-ft/ft} \]
   \[ M_w = 12.1 \text{ k-ft/ft} \]
   \[ H = 22'' = 1.83 \text{ ft} \]
   \[ \ell = 3.5 \text{ ft} \]
   \[ L_{cr} = 6.31 \text{ ft} \]
   \[ R_w = 126.3 \text{ kips} \]
   \[ & R_w = 114 \text{ kips} \]

INTERIOR WALL STRENGTH = 114 kips

114 > 100 kips → MEETS MASH TL-4 STRENGTH
END SECTION
YIELD LINE CALC.

MC VALUES: THE END SECTION (x 4 ft) HAS SAME
STIRRUPS, BUT AT CONDENSED SPACING.
AVERAGE SPACING = 6" OR 1/2 OF PREVIOUS INT. SPACING.
* THUS, FOR SIMPLICITY, JUST MULTIPLY INT. MC
VALUES BY 2.

MW VALUES: * SINCE END SECTION BENDING ONLY HAS THE
SINGLE YIELD LINE, MUST USE RESISTANCE I
STRENGTHS.

\[
\begin{align*}
\text{1:} & \quad M_c = 27.1 \times 2 = 54.2 \text{ k-ft/ft} \\
\text{2:} & \quad M_c = 18.3 \times 2 = 36.6 \text{ k-ft/ft} \\
M_w &= 17 \text{ k-ft/ft}, \quad M_w = 12.1 \text{ k-ft/ft} \\
\end{align*}
\]

* ALL H, L VALUES ARE THE SAME.

END SECTION YIELD LINE EQUATIONS.

\[
\begin{align*}
L_{cr} &= \frac{h}{2} + \sqrt{\left(\frac{h}{2}\right)^2 + H(M_b + M_w H)/M_c} \\
R_w &= \frac{M_b}{L_{cr} - \frac{h}{2}} + \frac{M_w H}{L_{cr} - \frac{h}{2}} + \frac{M_c L_{cr}^2}{H(L_{cr} - \frac{h}{2})}
\end{align*}
\]

RESULTS

\[
\begin{align*}
\text{1: FULL SECTION} & \quad \text{2: TOP SECTION} \\
& \quad \text{L}_{cr} = 4.1 \text{ ft} \quad \text{L}_{cr} = 3.8 \text{ ft} \\
R_w &= 156 \text{ kips} \quad R_w = 152 \text{ kips} \\
\Phi R_w &= 140 \text{ kips} \quad \Phi R_w = 136 \text{ kips}
\end{align*}
\]

END SECTION STRENGTH = 136 KIPS
- This barrier has enough capacity for T2-4 of mask.

- Stirrup spacing in the end sections is very uneven. I would aim for a consistent 6" spacing. May have to change a few locations to avoid guardrail connection bolts.

- To ensure end section strength, I would use 6" stirrup spacing within 5 ft from end.

- End section reinforcement/stirrups should be used at all expansion joints, or any other discontinuity. Current configuration only within 1 ft of joint.

- Barrier width: reinforcement could be optimized to reduce materials and still obtain Qw = 100 kPa.

- Remove deflection joint from standard drawings. Not good for strength.